

EXTENDED CURRICULUM

Learning outcomes

By the end of this unit you should be able to understand and use:

- set notation such as $n(A)$, $\in$, $\notin$, $\mathcal{E}$, A' , $\cap$, $\cup$, $\subseteq$
- Venn diagrams and appropriate shading of well-defined regions
- calculations involving missing regions in sets
- exponential sequences
- subscript notation for sequences

1.07 Further sets

Inclusion in a set

The symbols $\in$ and $\notin$ indicate whether or not an item is an element of a set.

1.13 Worked example

Set $A = \{2, 5, 6, 9\}$. Describe which of the numbers 2, 3 or 4 are elements and which are not elements of set A .

Set A contains the element 2, therefore

$$2 \in A.$$

Set A does not contain the elements 3 or 4, therefore

$$3, 4 \notin A.$$

The complement of a set

The complement of a set A, A' , is the set of elements of $\mathcal{E}$ which do not belong to A .

1.14 Worked example

If $A = \{3, 5\}$ and the universal set is the odd numbers from 1 to 11, write down the complement of A .

$$A' = \{1, 7, 9, 11\}$$

The empty set

This is a set that contains no elements, and is denoted $\emptyset$ or $\{\}$. For example, for some readers, the set of people who wear glasses in their family will have no members. The empty set is sometimes referred to as the null set.

Subsets

If all the elements of a set A are also elements of a set B then A is said to be a **subset** of B , $A \subseteq B$.

Every set has at least two subsets, itself and the null set.

TIP

The number of subsets can be found by using the formula 2^n , where n = number of elements in the set.

1.15 Worked example

List all the subsets of $\{a, b, c\}$.

The subsets are $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and $\{a, b, c\}$, because all of these elements can occur in their own right inside the main set.

Proper subsets

Proper subsets will contain all elements except the whole set and the null set.

1.16 Worked example

Set $A = \{2, 3, 5\}$, find:

- a the subsets b the proper subsets

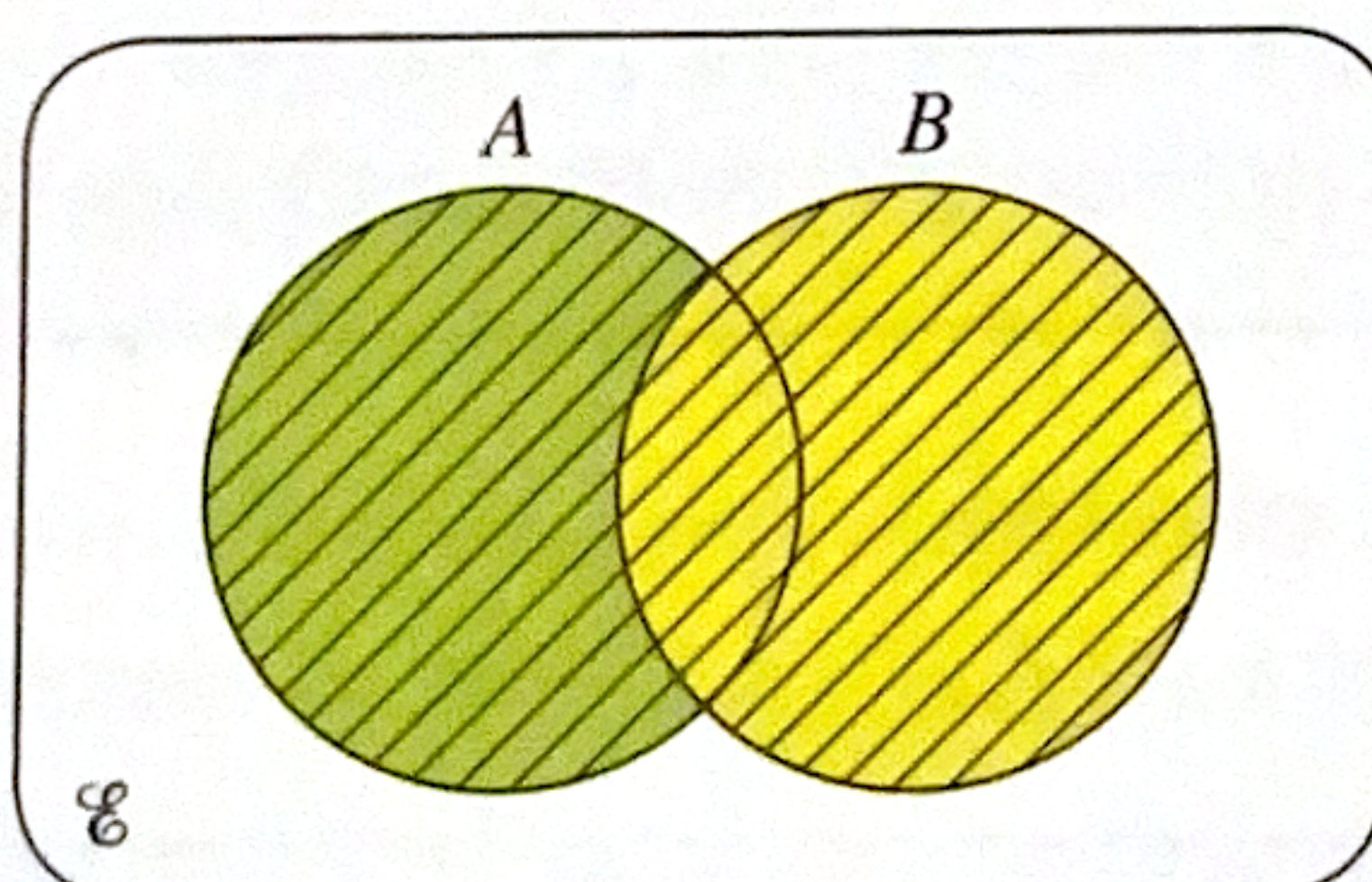
a $\{ \}$, $\{2\}$, $\{3\}$, $\{5\}$, $\{2, 3\}$, $\{2, 5\}$, $\{3, 5\}$, $\{2, 3, 5\}$,
note $\{ \}$ is another way of writing the empty set

b $\{2\}$, $\{3\}$, $\{5\}$, $\{2, 3\}$, $\{2, 5\}$, $\{3, 5\}$

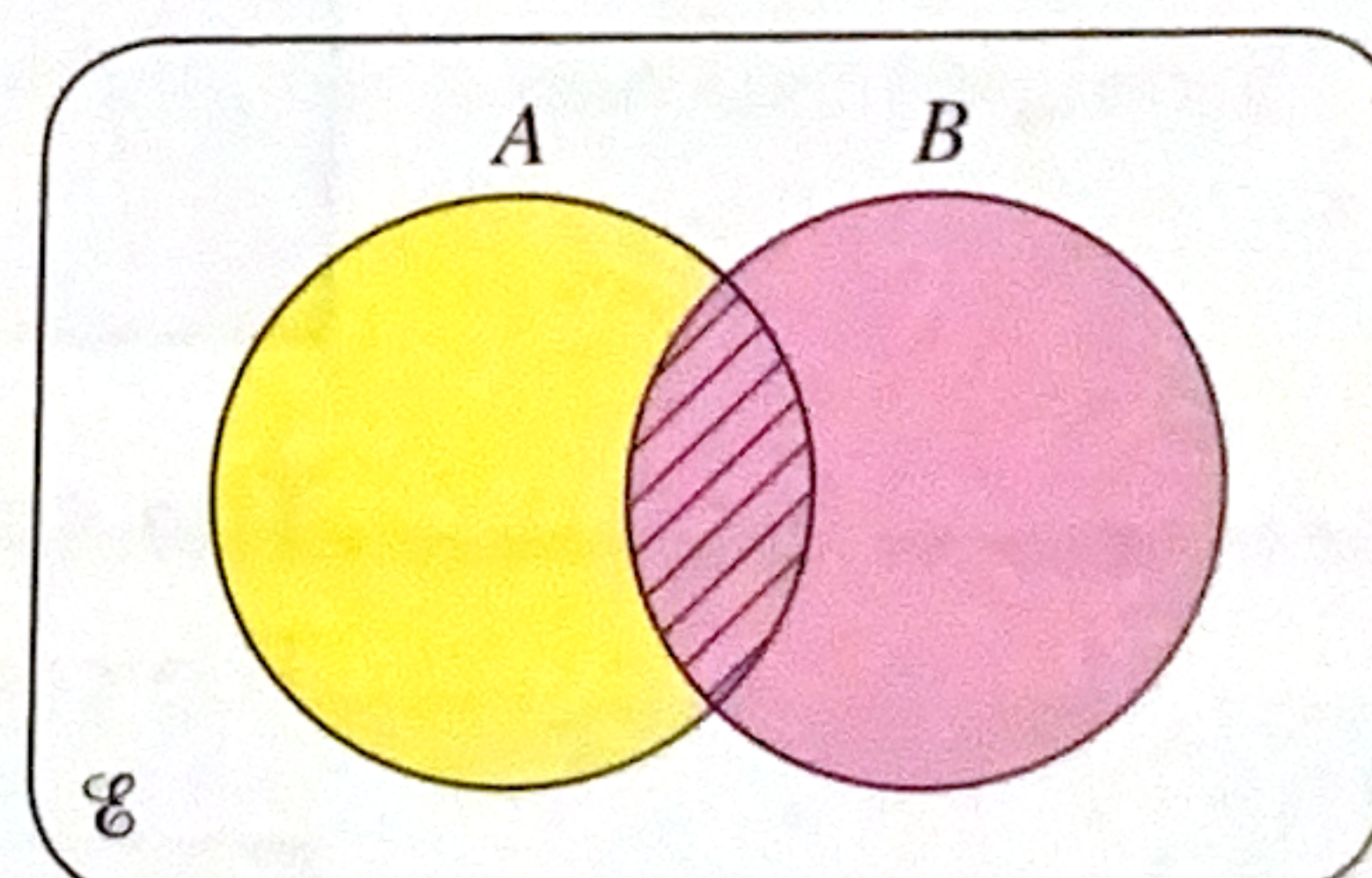
Venn diagrams

Set problems may be solved by using Venn diagrams. The universal set is represented by a rectangle and subsets of this set are represented by circles or loops. Some of the definitions explained earlier can be shown using these diagrams.

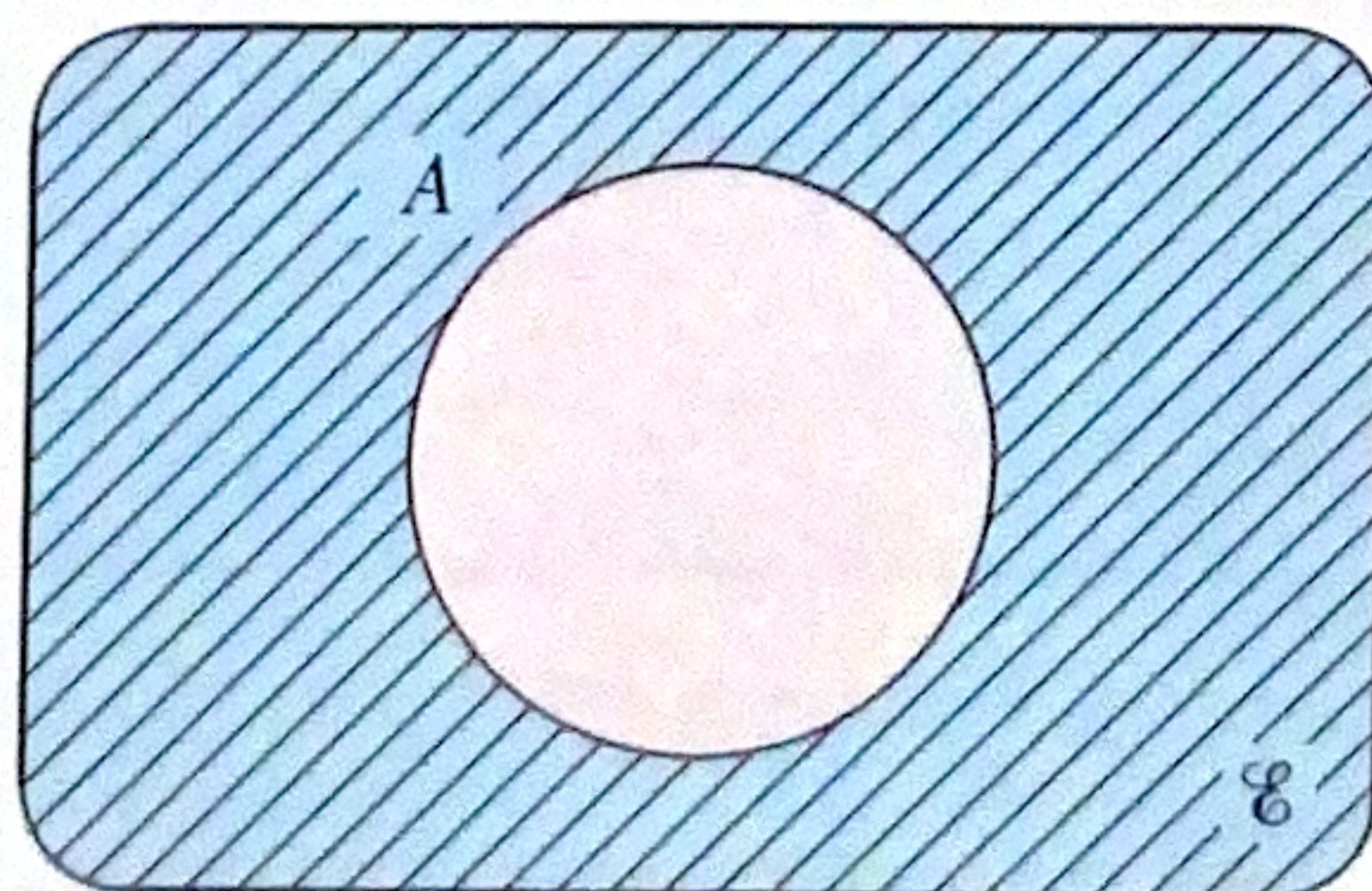
$A \cup B$



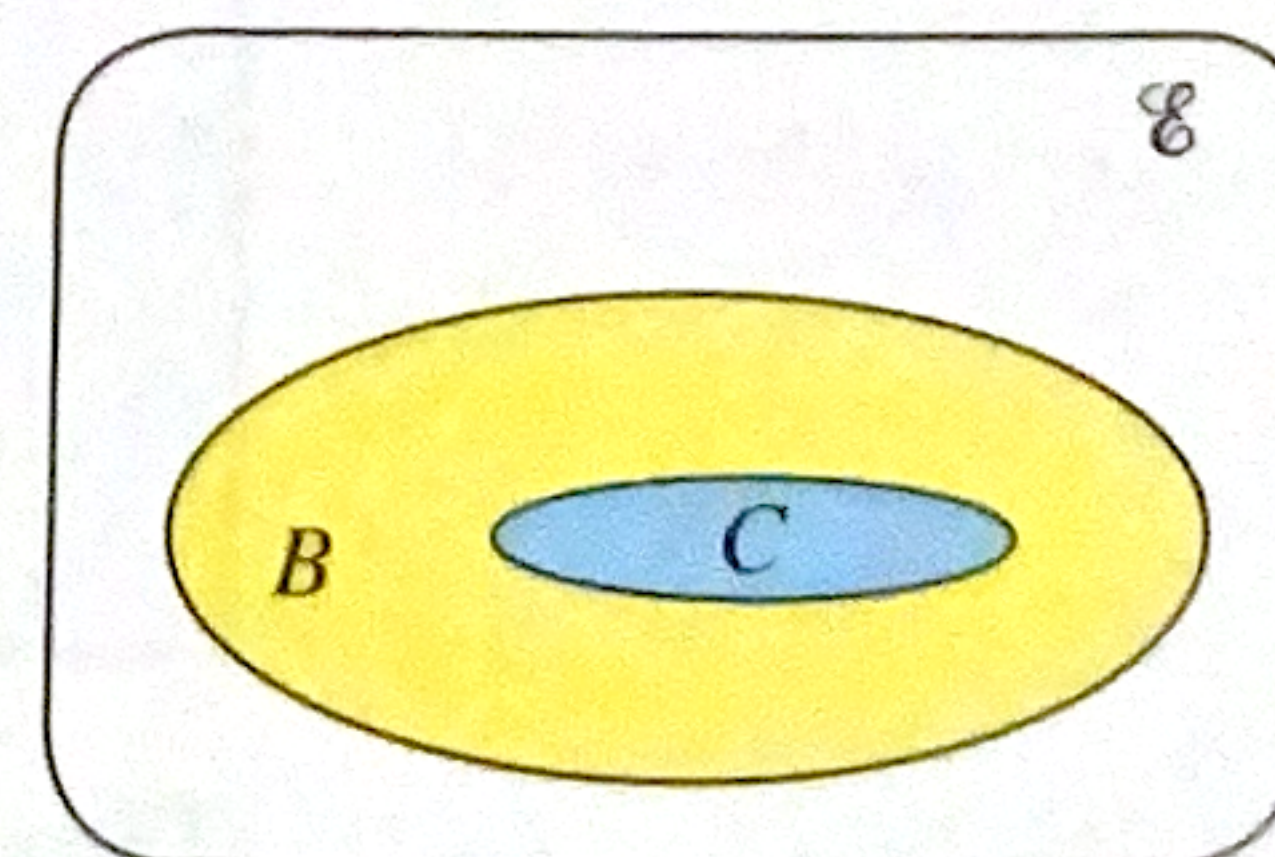
$A \cap B$



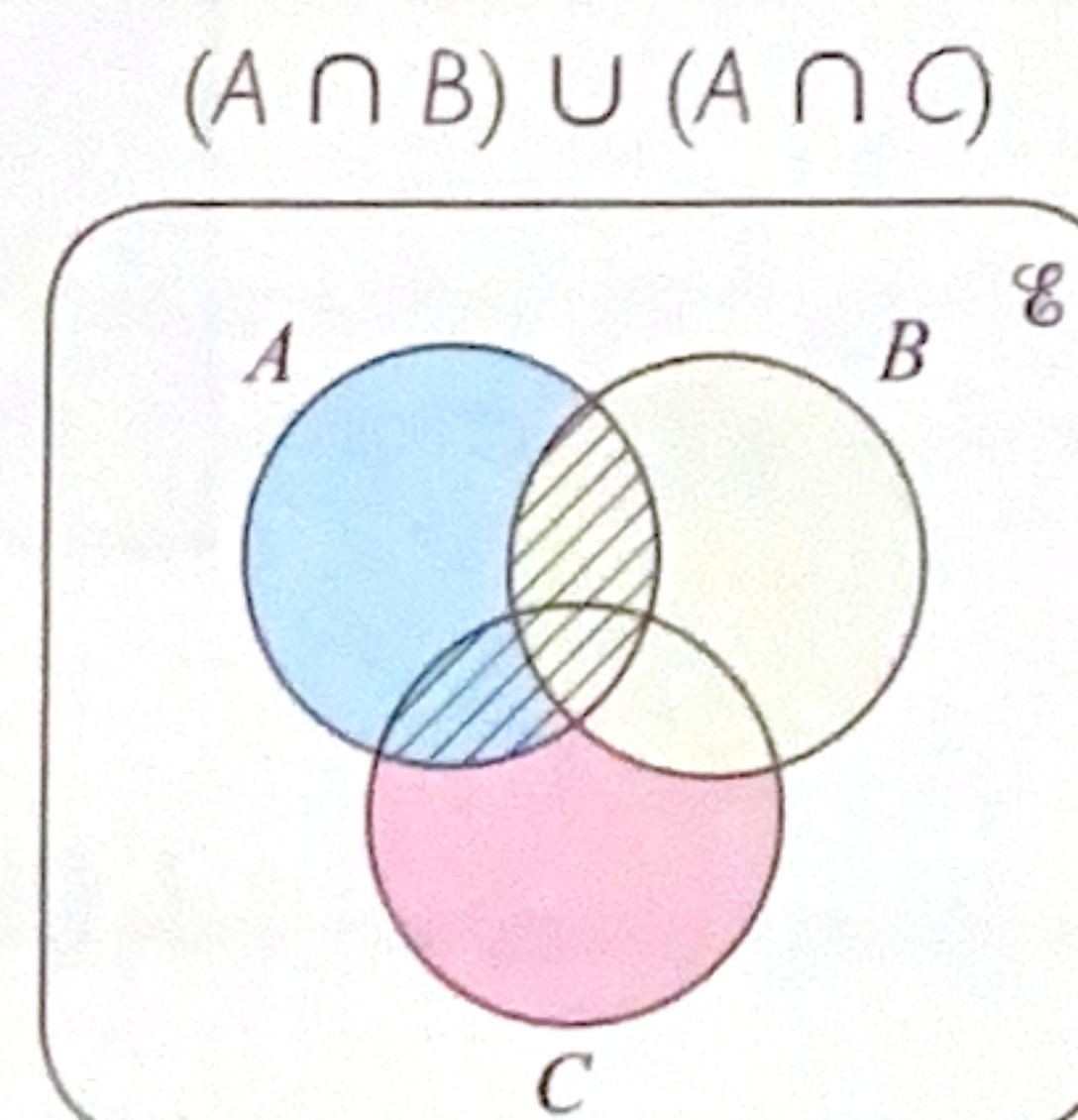
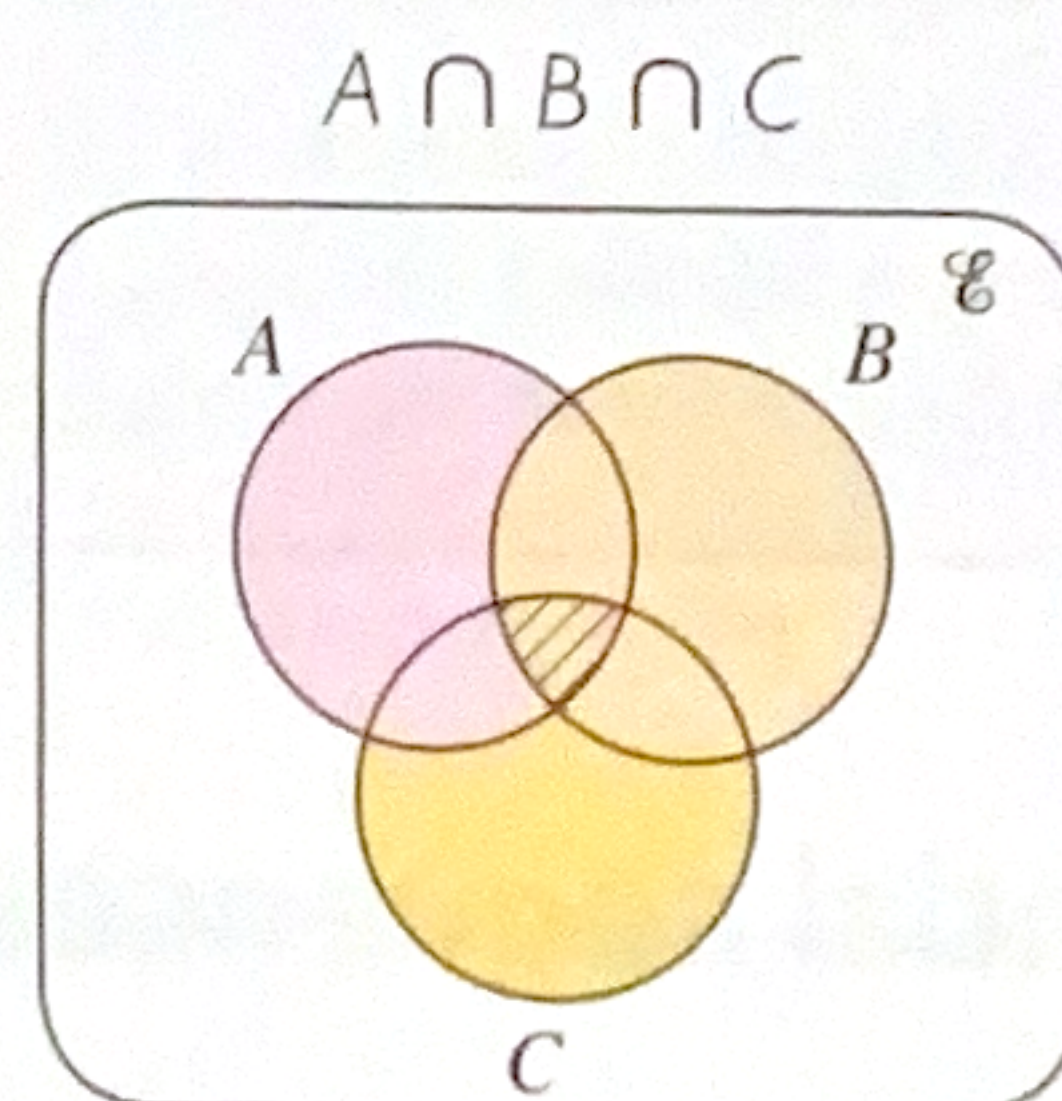
A'



$C \subset B$



Some more complex Venn diagrams:



Obviously there are many different arrangements possible with these diagrams but now let us try some more difficult problems.

1.17 Worked example

If $A = \{3, 4, 5, 6\}$, $B = \{2, 3, 5, 7, 9\}$ and $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, draw a Venn diagram to represent this information. Hence write down the elements of:

a A'

b $A \cap B$

c $A \cup B$

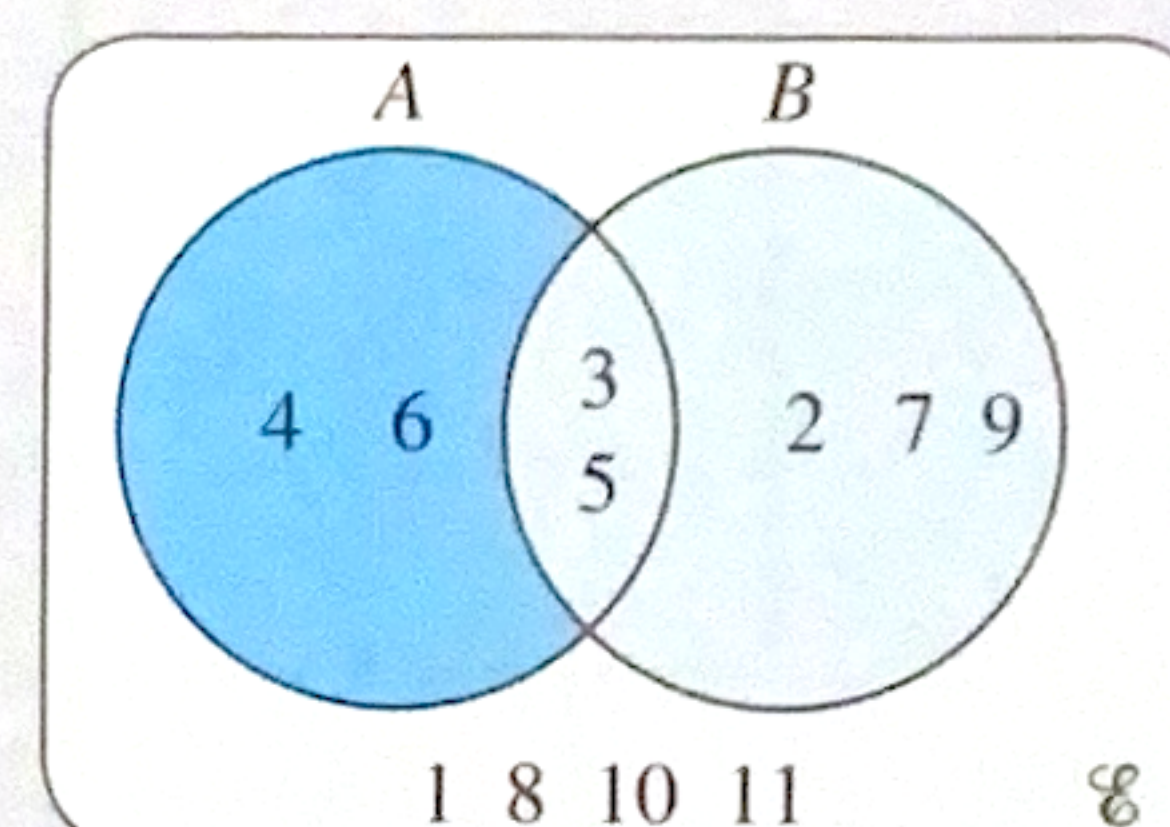
We only have 2 sets (A, B), so there are two circles inside the universal set.

From the Venn diagram it can be seen that:

a $A' = \{1, 2, 7, 8, 9, 10, 11\}$

b $A \cap B = \{3, 5\}$

c $A \cup B = \{2, 3, 4, 5, 6, 7, 9\}$



Problems with the number of elements of a set

For two intersecting sets A and B we can use the rule:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This formula can be used for any problem involving two sets; here is an example.

1.18 Worked example

In a class of 25 members, 15 take history, 17 take geography and 3 take neither subject. How many class members take both subjects?

Let H = set of history students, so $n(H) = 15$.

Let G = set of geography students, so $n(G) = 17$.

Also, $n(H \cup G) = 22$ (since 3 students take neither subject).

Let x represent the number taking both subjects.

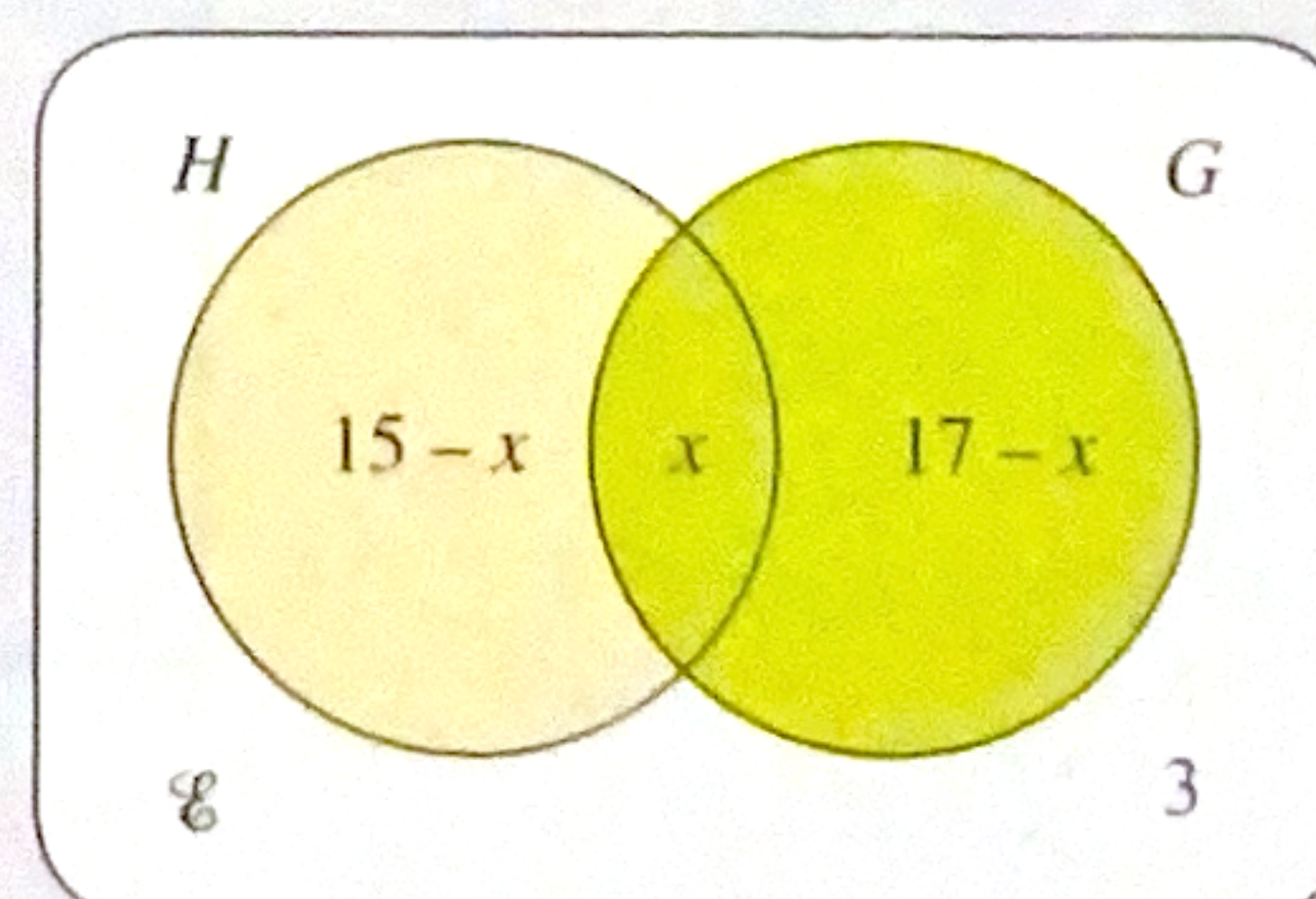
Now we can draw the Venn diagram.

Using the formula:

$$n(H \cup G) = n(H) + n(G) - n(H \cap G)$$

$$22 = 15 + 17 - x$$

$$\text{So } x = 10.$$



Hence the number taking both subjects is 10.
 The history-only region is labelled $15 - x$.
 This is because a number of these students also take geography.

1.08 Exponential sequences

These take the form a^n , where a is a positive integer. For example, $2^n, 2^{n+1}, 3^n$.

Here is a table showing some typical exponential sequences:

n	1	2	3	4	5	6
2^n	$2^1 = 2$	$2^2 = 4$	8	16	32	64
3^n	$3^1 = 3$	$3^2 = 9$	27	81	243	729

Notice that the differences between successive terms are steadily increasing, but they represent powers of 2, 3, etc.

1.19 Worked example

Write down the first five terms of the sequence $2^{n+1} + n^2$.

$$n = 1, 2^{(1)+1} + (1)^2 = 5$$

$$n = 2, 2^{(2)+1} + (2)^2 = 12$$

$$n = 3, 2^{(3)+1} + (3)^2 = 25$$

$$n = 4, 2^{(4)+1} + (4)^2 = 48$$

$$n = 5, 2^{(5)+1} + (5)^2 = 89$$

So the first five terms are 5, 12, 25, 48, 89.

1.09 Subscript notation $U_1, U_2, \dots, U_n$

We can represent the terms using subscript notation as shown above. The first term is U_1 , second term U_2 , etc... the n th term is U_n .

1.20 Worked example

In an arithmetic sequence, the n th term is $3n + 2$. Write the first three terms of the sequence.

$$U_1 = 3(1) + 2 = 5$$

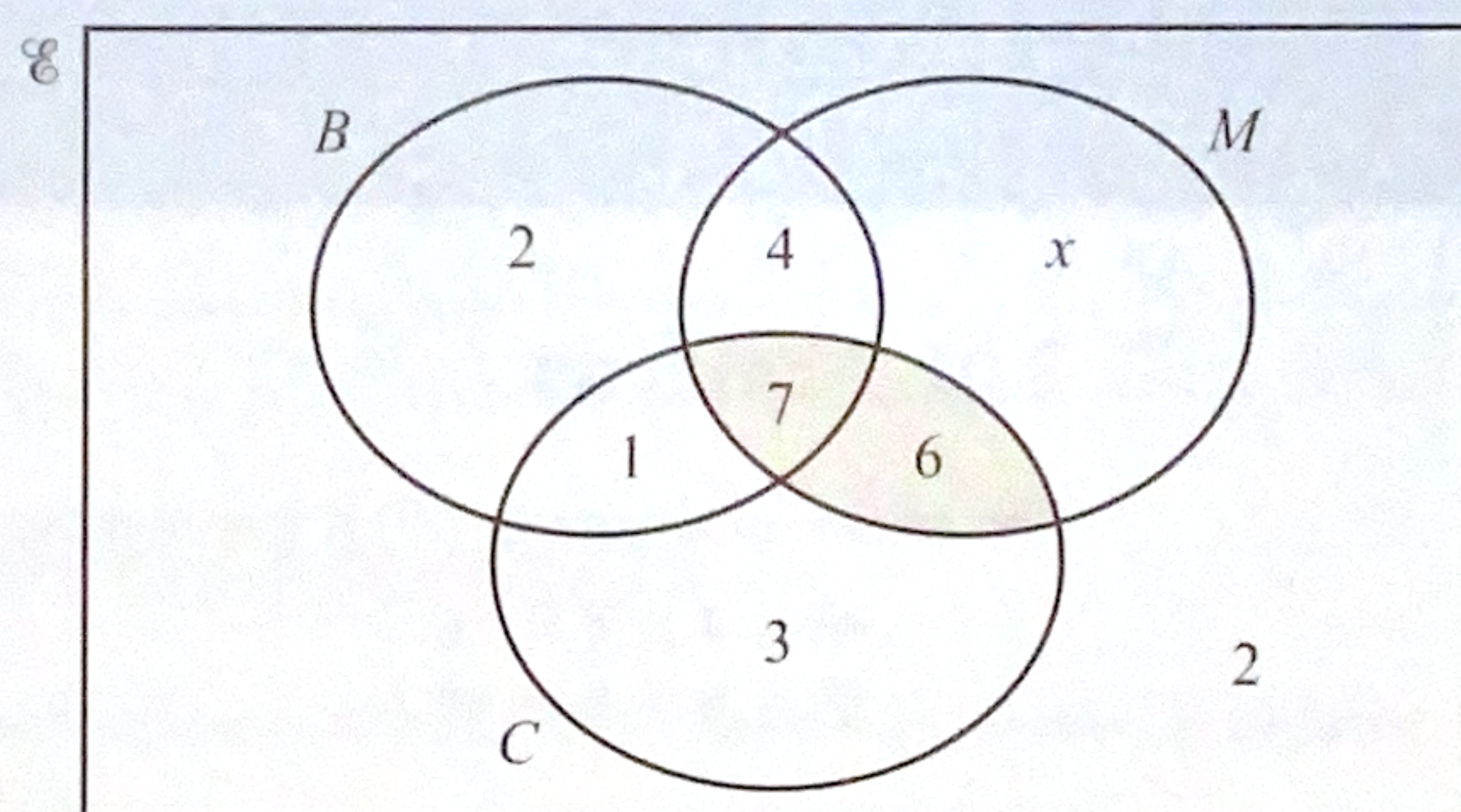
$$U_2 = 3(2) + 2 = 8$$

$$U_3 = 3(3) + 2 = 11$$

- $\in, \notin$
An element is or is not an element of a set.
- A'
The complement of a set: the elements in the universal set but not in set A .
- $\subseteq$
The subset of a set: another set made from some or all of the elements in the main set.

Exam-style questions

- 1.05 30 students were asked if they had a bicycle (B), a mobile phone (M) and a computer (C). The results are shown in the Venn diagram.



- Work out the value of x .
- Use set notation to describe the shaded region in the Venn diagram.
- Find $n(C \cap (M \cup B)')$.
- A student is chosen at random.
 - Write down the probability that the student is a member of the set M' .
 - Write down the probability that the student has a bicycle.
- Two students are chosen at random from the students who have computers.
Find the probability that each of these students has a mobile phone but no bicycle.

Source: Cambridge IGCSE® Mathematics 0580 Paper 41 Q4 June 2015.